ISSN: 0975 -8542

Journal of Global Pharma Technology

Available Online at: www.jgpt.co.in

RESEARCH ARTICLE

Characteristics of Modes in Photonic Crystal Fibers

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Abstract

In this article, both the vector effect index method (VEIM) and finite element method (FEM) are exploited to calculate the effective index of FSM for the solid-and porous core PCF. For the solid-core, the EIM applies one time to estimate the equivalent cladding index. Then, the conventional characteristics' equation will apply to construct the n_{eff}. For porous-core PCF, the EIM applies two times one for the core and other for cladding. Next, also the conventional characteristic equation will apply to explain the n_{eff} at the porous-core PCF.

Keywords: *Finite element method, Porous core PCF, Vector effect method.*

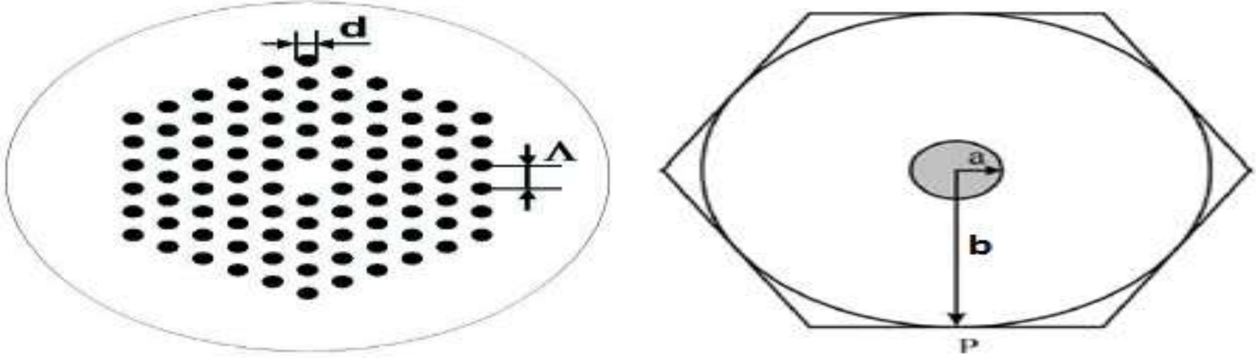
Introduction

One of the most important and useful applications at photonic crystals is the design of novel waveguides such as solid-core PCF and porous-core PCF. PCFs are single material optical fibers with an array of periodic air holes across the cross-section running down its entire length. By leaving a single lattice sit without an air hole, a localized region acts as the waveguide cone in which light can trapped along the axis of the fiber. The photonic crystal with periodicity in transverse direction supports mode propagating along the longitudinal direction.

These modes are referred as SFM because they are infinitely extended in the transverse direction [1]. In the effective index method (EIM), a single material having refractive index equal to the modal index at fundamental SFM replaces the photonic crystal cladding. An effective cladding index will analyze for an understanding of PCF properties. The sharp contrast between this effective cladding index and the index of core material obtains anomalous properties of these fibers, whom the effective index model used to study mode-field radius, PCF design, and even to estimate field distributions [2].

Effective Index Method

There are many techniques used to analyze the PCFs, such as the effective index method (EIM), the localized basis function method, the finite difference method (FDM), finite element methods (FEM), the plane wave expansion method (PWM). The EIM was the first model successfully used to construct the properties of PCFs. This method has been focused to calculate the dispersion of PCFs that propagate light by modified total internal reflection, where modeling of PCFs with a complicated index profile using numerical methods is computationally intensive [3]. The EIM includes the scalar EIM (SEIM) and the fully vectorial EIM (VEIM), in which the VEIM is better because the SEIM is only applicable for weakly guiding approximation and its applicability reduces when apply to PCFs with a large air-filling fraction, whereas the VEIM has no such limitation. In general, the VEIM is still has less accurate than many numerical methods such as FEM and FDM. In addition, the definition of the radius of the unit cell in calculating the effective cladding index, n_{FSM}, and the fiber core of the corresponding approximate step-index fiber remain ambiguous and different values are used in literature [4]. The accuracy of the VEIM can be enhanced by selecting appropriate values for the effective core radius when PCFs have different fiber parameters. The accuracy of VEIM can be tested by comparing its results with accurate numerical results obtained by FEM using COMSOL software. The cross-section and characteristics parameters of a PCF are schematically shown in Fig. (1a), where a missing air hole forms the fiber core, Λ is the hole spacing or pitch and $d = 2a$ is the air hole diameter. The effective index of cladding is determined by approximating the hexagonal unit cell as an equivalent circular unit cell of outer radius R and inner radius a by keeping constant air filling function as shown in Fig. (1b). The VEIM typically uses a fixed value of r_c in modeling PCFs with different relative air hole diameters, e.g. $r_c = \Lambda / \sqrt{3}$ or $r_c = 0.625\Lambda$. More recently, there have been efforts in improving the VEIM further by taking into consideration the fact that the effective core radius r_c should vary a function of the relative air hole diameter d/Λ or even also a function of the relative wavelength λ / Λ [5].



(A) (B)

Fig.1: a) the characteristics cross-section, b) the hexagonal unit cell [3]

In VEIM, both n_{FSM} and n_{eff} are obtained using a fully vectorial approach. The procedure for calculating the modal index of the fundamental mode is similar to the previous method. One starts with the wave equation for the electromagnetic components E_z and H_z as [6]:

$$\left[\nabla_t^2 + (w/c)^2 n^2 - \beta^2\right]\begin{bmatrix} E_z \\ H_z \end{bmatrix} = 0 \qquad (1)$$

From Maxwell's equations, one can get the expressions of transverse components of E_z and H_z (in the fiber core and cladding), and then by applying the continuity conditions at the interface of hole and silica region, the fields will be [7]

$$\psi_1 \sim I_\ell(WR) \qquad \text{core region} \qquad (2a)$$

$$\psi_2 \sim Y_\ell(u)J_\ell(UR) - J_\ell(u)Y_\ell(UR) \qquad \text{cladding region} \qquad (2b)$$

Where $\psi_{1,2}$ represents electric or magnetic field components.

If one define $P_\ell(UR) = Y_\ell(u)J_\ell(UR) - Y_\ell(UR)J_\ell(u)$, then the characteristic equation for determination n_{FSM} is of the form [8]

$$\left[\frac{P_\ell'(U)}{UP_\ell(U)} + \frac{I_\ell'(W)}{WI_\ell(W)}\right]\left[n_s^2 \frac{P_\ell'(U)}{UP_\ell(U)} + n_{air}^2 \frac{I_\ell'(W)}{WI_\ell(W)}\right] = n_{FSM}^2 \ell^2 \left[\frac{1}{U^2} + \frac{1}{W^2}\right]^2 \qquad (3)$$

Where U, W, u as defined in the previous case. Since $\beta_{FSM} = kn_{FSM}$ denotes the propagation constant of the fundamental mode propagating in an infinite self-similar hexagonal lattice without any defect, one may be considered $\ell = 1$ in the above equation while determining n_{FSM}. Once n_{FSM} is obtained, the propagation constant of the fundamental guided mode, $\beta_{eff} = kn_{eff}$ and hence the corresponding effective index n_{eff} of the PCF may be calculated using the following characteristics' equation [8].

$$\left[\frac{J'_\ell(U_{eff})}{U_{eff}J_\ell(U_{eff})}+\frac{K'_\ell(W_{eff})}{W_{eff}K_\ell(W_{eff})}\right]\left[n_s^2\frac{J'_\ell(U_{eff})}{U_{eff}J_\ell(U_{eff})}+n_{FSM}^2\frac{K'_\ell(W_{eff})}{W_{eff}K_\ell(W_{eff})}\right]=n_{FSM}^2\ell^2\left[\frac{1}{U_{eff}^2}+\frac{1}{W_{eff}^2}\right]^2 \quad (4)$$

Where U_{eff}, W_{eff} as defined in the previous case. This equation is evaluated for an equivalent single mode fiber with the indices n_s, n_{FSM} for core and cladding, respectively. Also, by setting $\ell = 1$ in this equation, one can obtain the effective index of the fundamental propagating mode. Albeit expository techniques are not computationally escalated and take lesser time, yet much of the time of PCF structure investigation they do not yield extremely exact outcomes. Therefore, scientists turned to numerical techniques for exact and right examination of different plans of PCFs. In numerical counts and investigations, a financially accessible software bundle from COMSOL Multi-physics actualizing the FEM has been utilized

Finite Element Method

Finite Element Method is a numerical method that proposes an approximate solution to the boundary value problem [9]. Starting from 1940's the FEM has a history of solving a boundary value problem in mathematics and physics. This method was initially focused on air craft design. Later it was adopted by the civil engineers for structural design [10]. Today the method is extended to many other areas of physics and engineering. An approximate solution to any complex engineering problem can be reached by subdividing the problem into smaller, more manageable (finite) elements.

Using finite elements, solving complex partial differential equations that describe the behavior of certain structures can be decreased to a set of linear equations that can be easily solved using the standard techniques of matrix algebra [10].FEM is a unique numerical method which addresses problems in areas of physics and engineering that include fluid mechanics, mechanics of materials, chemical reactions, semiconductor devices, electromagnetics, optics, quantum mechanics, acoustics, etc.

The key features of FEM are [11]: piecewise approximation of continuous the field gives good precision even with the simple approximating functions, using computational time and resources we can improve the precision just by increasing the quantity of components, FEM analysis approximates the element of small sizes and different shapes. This eases the application of complex the shape of different materials and different boundary conditions.

It covers both linear and nonlinear problems. Local approximation generates sparse the system of the equation that helps to solve the problem having a large number of modal unknown. FEM substitute the continuous function by a number of discrete sub domains. The unknown function in the sub domain is represented by simple interpolation functions with unknown coefficient [9].

That approximates the original boundary value system by a finite number of unknown coefficients. Ritz variation or Galerkin procedure convert the boundary value issue to a finite number of algebraic equations. Solving these algebraic equations led us to find the unknown coefficients. With the sparsity of the coefficient matrices, FEM exhibits pleasing characteristic of computational economy in numerical modeling [10].

The success of FEM in electromagnetics can be largely attributed to their great adaptability and flexibility, which allow the treatment of geometrically complex structures with inhomogeneous, anisotropic or even nonlinear materials [12]. In summary, the boundary value problem solution using FEM should have the following steps [13]: discretization of the domain, the proper selection of the interpolation function, formulation of the system of algebraic equations using Ritz or Galerkin method, and the solution of system of algebraic equations.

Results and Discussion

In this paper, a n_{eff} was calculated for two types of PCF: solid core and porous core, using two methods: the analytical method VEIM and numerical method FEM. Where the numerical method adopted the use of the COMSOL program, which we will shorten its work in terms of our search in appendix A. Knowing that the accuracy of the results using this program depends on the settings of the method, as follows: if the used mesh is soft, then the results will improve, but this increases the expense of the time necessary for implementation. On the other hand, the analytical method relied on the type of fiber used, where in the solid core; Eq. (3) was used to calculate n_{FSM} and then applied Eq. (4) to calculate n_{eff} for the entire fiber. Whereas in the porous core type Eq. (3) was used two times for the calculation of and then again and then using Eq. (4) for calculating n_{eff} for the entire fiber. Fig. (2) Represents the relationship between the wavelength and the effective refractive index using several values of the diameter of the hole with the plotting n_{silica} for all cases for the purpose of comparison.

The first column of the form represents the calculations in FEM method using COMSOL program and the second column represents the calculations by VEIM. In general, it appears that the effective refractive index decreases with increasing the wavelength, increasing d means distancing n_{eff} from n_{silica} and vice versa. Fig. 3 represents the relation of V_{eff} and $\frac{d}{\Lambda}$ using several values of Λ and several wavelengths. The broad red line here also points to the boundary between a single mode and multi-modes regions. From the figure, we can see that the fact that at $\lambda = 1.6\mu m$, for example, in $\Lambda = 2$ does not allow multi-modes at all, but the situation is different at $\Lambda = 3$. In general, a decrease of λ tends to make all Λ values achieve multi-modes, particularly at high d values. Generally, the multi-mode property is affected by the difference between the refractive indices of parameters of the core and the cladding in addition to the wavelength used. Fig.(4) shows the relationship of n_{eff} as a function of wavelength when the core is porous, where the first column represents the numerical method and the second column represents the VEIM method and rows represent several values of the diameter of the holes of the cladding and represent the colors using several diameters for the holes to the core.

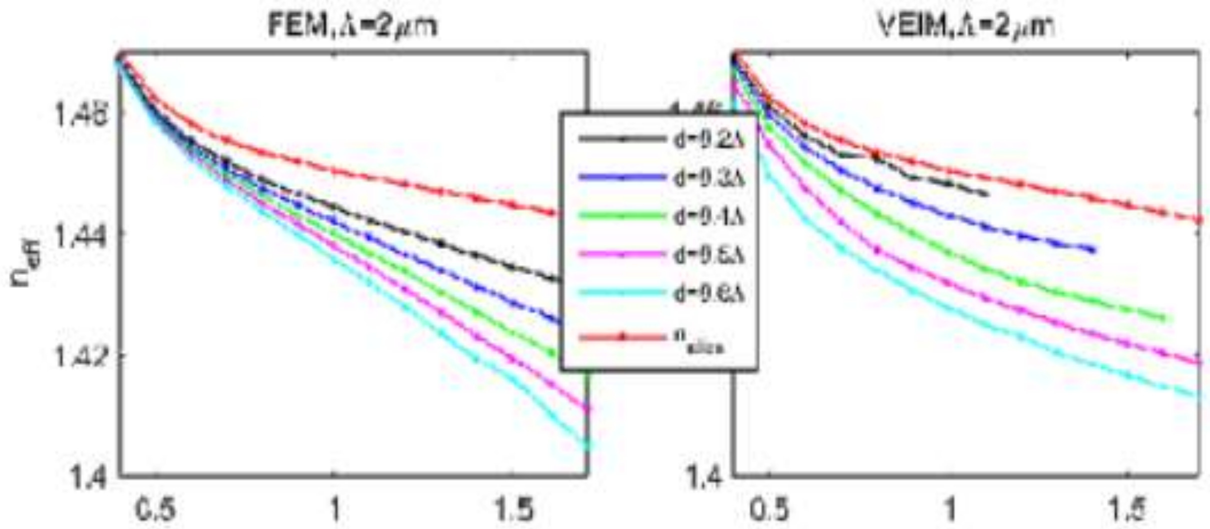


Fig.2: n_{eff} **as function for wavelength, where the different colors refer to different the value at** d **and** $n_{sillica}$

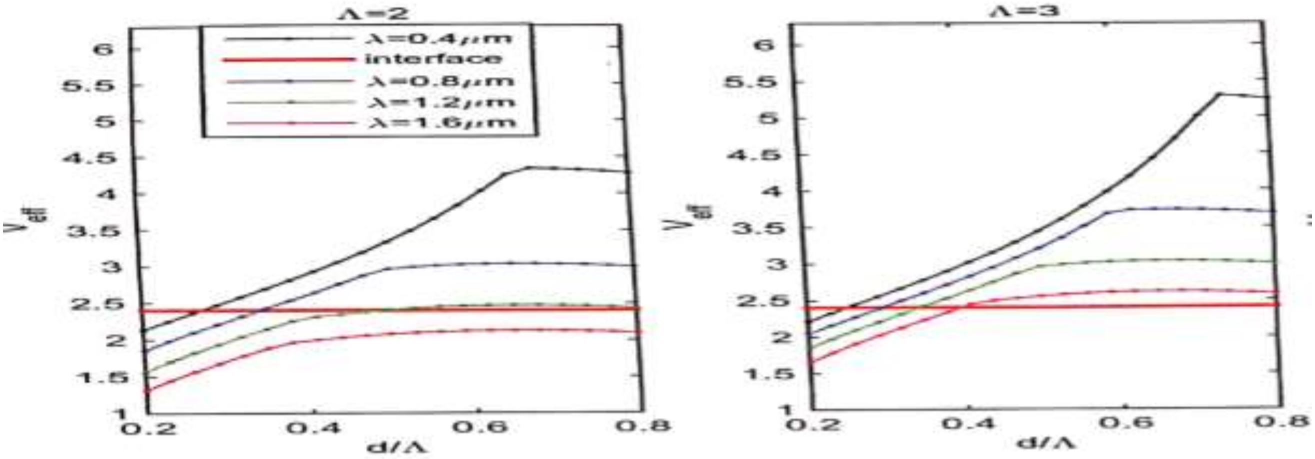


Fig.3: relation of V_{eff} and $\frac{d}{\Lambda}$ using several values of Λ and several wavelengths

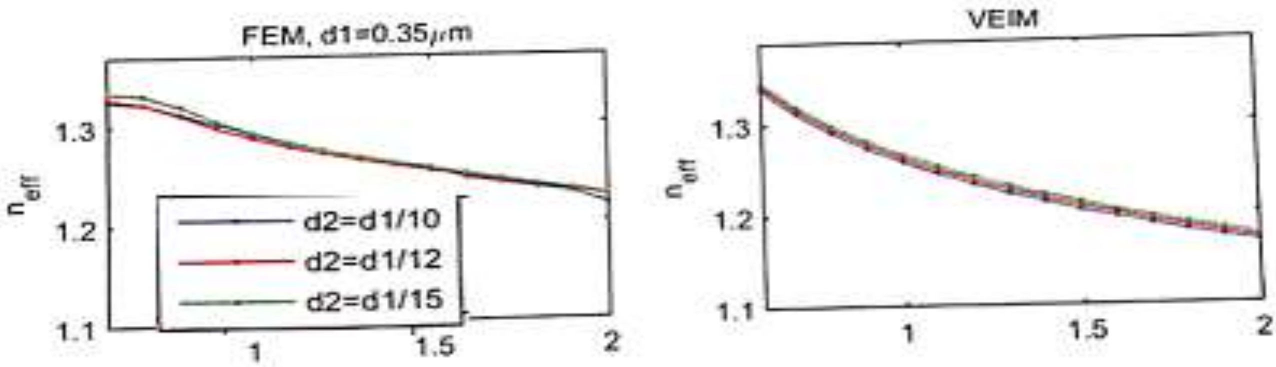


Fig. 4: n_{eff} is a function for wavelength when the core is porous, for different d_2 values

In general, n_{eff} is reduced with increasing wavelength, but increasing d_1 will low ring the curve n_{eff} for all cases. When d_1 is lower, the d_2 change does not significantly affect the results as long as its values followsd_1. Comparing to the results in two columns, it is clear that there is a significant difference between the two methods caused by the application of VEIM three times that applied to the cladding, the core and the fiber. Because of the huge of calculations and the possibility of adding errors every time, we apply VEIM, the frequency of their application will inevitably increase the amount of error.

FEM also adds errors because the presence of a number of small holes in the core forces us to use a super-precision mesh in the COMSOL program. The finer mesh will added huge computation and will cause some additional errors. Fig. (5) Represent V_{eff} as a function of wavelength using several values of d_2 ,Λ_1 ,Λ_2 , where $\Lambda_2 = \frac{\Lambda_1}{2}$. The broad red line represents the interface between SMF and MMF. It is clear that $\Lambda_1 = 0.5\mu m$ does not provide any opportunity for the emergence of more than one mode and with the increasingΛ_1, the chances of the emergence of more than mode increases, especially at the shorter values of λ. The d_2 values indicated on the figure can lead us to conclude that a decreasing of d_2 will increase the chances of the emergence of higher-order modes. This can be explained with the basis that a decreasing d_2 will lead to the proximity of the porous type to solid type and hence the tendency to MMF. Fig.(6) Obtain n_{FSM} for core and cladding and V_{eff} as functions of the wavelength using several values to Λ_1 where$\Lambda_2 = \frac{\Lambda_1}{2}$. In the first column, we plot the refractive index of the core, which changes little with a change in the value of d_2, In general, the refractive index of the core value around the refractive index of silica as long as the holes are very small.

The second column represents the reactive index of the cladding. We see that the large difference caused by the increase of d_1, where the refractive index of the cladding tends to decrease significantly with the large value d_1 as long as this causes an increasing of air ratio with respect to silica in the cladding. On the other hand, an increasing of Λ_1 also means a decreasing of air ratio and therefore raising the refractive index. The third column, shows that the tendency to the MMF area is associated with an increasing of Λ_1 and an increasing of d_1, where we have previously indicated that this means that the fiber will approach the solid-core fiber and thus increasing the chances of the emergence of higher order modes.

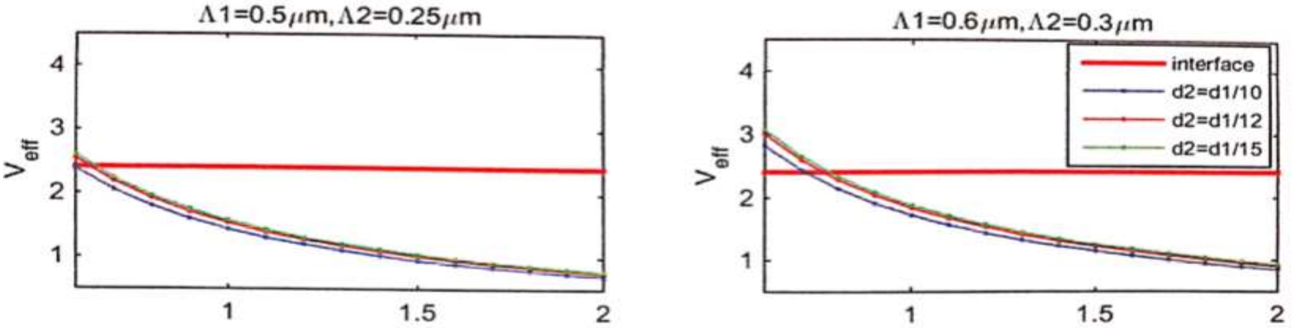


Fig.5: V_{eff} **as a function for wavelength using several values of** $d_2, \Lambda_1, \Lambda_2$.

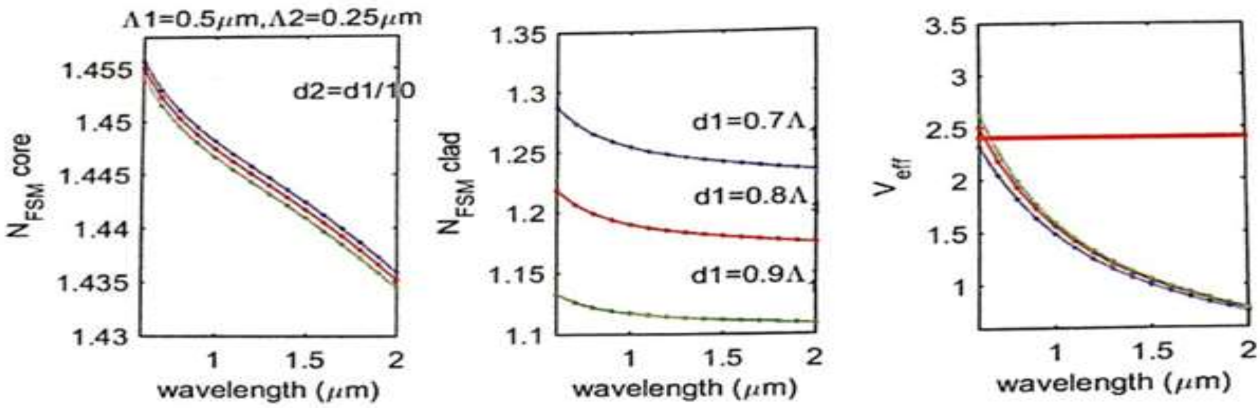


Fig.6: n_{FSM} **for core and cladding and** V_{eff} **as a function for wavelength for different** Λ_1 **value**

Conclusions

In the case solid-core PCF, the high fill rate within the perimeter of the cladding means approaching the case of normal fiber and low fill ratio caused a clear drop to curve n_{FSM}. In the case porous-core PCF, the ratio of filling within the perimeter of the cladding similar to the previous point, but the ratio of filling within the core region is very sensitive in determining the number of modes and determining the accuracy n_{FSM}. The method of analysis of VEIM is close to the results of FEM when it comes to high fill rates in the core and cladding regions. The analytical method VEIM sometimes fails to provide a solution because of the inability to obtain a solution for the characteristic equations.

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